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Lewis-Ransing correlation to optimally design the metal-mould heat transfer

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Abstract *Heat transfer across the metal-mould interface has been modelled by a generalized equation referred to as the Lewis-Ransing Correlation. It has been shown that the spatial as well as temporal variation of the interfacial heat transfer coefficient can be optimally designed to achieve a desired solidification pattern. The technique has been validated on two practical examples achieving a complex solidification pattern.*

1. Introduction

One of the major objectives of solidification analysis is to predict the presence of hot spots i.e. the locations in a casting which solidify last. The primary and most obvious phenomenon during solidification is the transfer of heat from the cooling metal to the mould.

Feeding design decisions such as insulation around a feeder, provision of chills, exothermic pads or die coating thickness in the case of gravity dies etc., are associated with an appropriate interfacial heat transfer coefficient value across the metal-mould interface. The objective of feeding design is to keep hot spots in the feeder i.e. to eliminate the shrinkage porosity a casting. The reliability of a numerical analysis which is used to assist with this, largely depend on the heat transfer model used across metal-mould interface.

There has been great interest on experimental investigation (Anderson, 1995; Schmidt and Svensson, 1994) as well as on numerical modelling research to understand the complexity of the heat transfer across the metal-mould interface. Until now, a large number of experiments have been reported which describe the behaviour of these interfacial heat transfer coefficients. In this paper, we have analyzed a variety of experimental results, as well as reviewing the current methods for considering the interfacial heat transfer coefficients. In the latter half of this paper, we have proposed a correlation to embody these coefficients into a simulation system followed by a demonstration of its use for the optimal feeding design application.

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2. Solidification analysis

The solidification analysis carried out is based on the heat conduction equation using the enthalpy method (Lewis *et al.*, 1996) to model phase change from liquid to solid. The problem domain is discretised with finite elements and, using a Galerkin weighted residual method, we obtain,

$$\mathbf{C}(T)\dot{\mathbf{T}} + \mathbf{K}\mathbf{T} = \mathbf{F} \tag{1}$$

where, $\mathbf{C}$ is the heat capacity matrix which depends upon temperature (to include latent heat effects in the relevant temperature interval), $\mathbf{K}$ is the conductivity matrix, and $\mathbf{F}$ is a load vector. Then

$$\mathbf{C}(T) = \sum_e \int_{\Omega_e} \rho c N_i^e N_j^e \, dxdy \tag{2}$$

$$\mathbf{K} = \mathbf{K_D} + \mathbf{K_H} + \mathbf{K_I} \tag{3}$$

$$\begin{aligned}\mathbf{K_D} &= \text{Diffusion part}\\ &= \sum_e \int_{\Omega_e} \left[\frac{\partial N_i}{\partial x} k \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} k \frac{\partial N_j}{\partial y}\right] dxdy\end{aligned}$$

$$\begin{aligned}\mathbf{K_H} &= \text{Convection to ambient part}\\ &= \sum_e \int_{\Gamma_{he}} N_i h_c N_j \, d\Gamma\end{aligned}$$

$$\begin{aligned}\mathbf{K_I} &= \text{Interface element contribution}\\ &= \int_l h N_i N_j \, dl \qquad i = 1,2 \;\; and \;\; j = 1,2\end{aligned}$$

$$\mathbf{F} = \sum_e \int_{\Gamma_{he}} N_i^e h_c T_\infty \, d\Gamma - \sum_e \int_{\Gamma_{qe}} N_i^e q \, d\Gamma \tag{4}$$

where, l is the length of the interface element, h is the interfacial heat transfer coefficient and N_i, N_j are standard finite element shape functions.

For the temporal discretisation an implicit Backward-Euler finite difference approximation was employed because of its inherent stability properties. The resulting system of equations was solved using a profile solver.

$$\left[\frac{\mathbf{C}_{n+1}(T)}{\Delta t} + \mathbf{K}_{n+1}\right](\mathbf{T}_{n+1}) = \left[\frac{\mathbf{C}_{n+1}(T)}{\Delta t}\right](\mathbf{T}_n) + (\mathbf{F}_{n+1}) \tag{5}$$

3. The Lewis-Ransing correlation

Ransing *et al.* (1993) observed that when the interfacial heat transfer coefficient variation is plotted against casting temperature at the interface, the variation follows a unique pattern. The decay in the interfacial heat transfer coefficient values is generally of an exponential nature. Later, Lewis and Ransing (1998) proposed a correlation to input these coefficients into a solidification program in a generalized way. The correlation (equation 6) has three coefficients which can be calibrated with the available experimental data and made available as a database. The casting interface temperature dependence in the correlation makes its use generic and it can easily be incorporated in a numerical code. The coefficients a_1, a_2 and a_3 vary within a range with upper and lower limits as indicated below. The correlation does not model air gap and the justification is detailed in our previous publication (Lewis and Ransing, 1998)

$$h = e^{a_1} * e^{-a_2/x**2} * \frac{1}{x^{a_3}} \tag{6}$$

$$x = \sqrt{2a_2/a_3} + \max(0,\ T_L - T)$$

where

- h: Interfacial heat transfer coefficient (W/m^{2}C)
- x: Intermediate variable
- a_1: Coefficient in the range of 10-15
- a_2: Coefficient in the range of 100-1,500
- a_3: Coefficient in the range of 0.2-1.2
- T_L: Liquidus temperature (C)
- T: Cast temperature at the interface

3.1 The Lewis-Ransing Correlation to optimize interfacial heat transfer in castings

In this section it will be demonstrated that the constants a_1, a_2 and a_3 in the Lewis-Ransing Correlation offers an ideal choice for the design variables for the optimal design of the interfacial heat transfer.

The objective is to enforce the directional solidification in the casting. Alternatively, if the hot spot is detected inside the casting then it needs to be moved into the feeder. The path for the directional solidification, which should be prescribed by the user, is also referred to as a feed metal flow path. Figure 1 shows two examples of the user-defined paths along which the directional solidification can be enforced. Later in the paper, both examples will be discussed in detail. The axisymmetric casting has one complex shaped prescribed feed metal flow path: 1-2-3-4-5. Point 5 is in the feeder and should solidify last and point 1 should solidify first. The freezing should continue along this path so that the point 2 feeds point 1, point 3 feeds point 2 etc. In the second example, which is a 2D casting, the objective is that both feeders should feed the casting along paths 6-5-4-3-2, 6-5-4-3-1, 14-13-11, and 14-13-12. Points 6 and 14 should be the last to solidify and 2, 1, 11 and 12 should solidify first to maintain the directional solidification. It can be seen that this example requires a more complex feed metal flow path distribution. However, for a complex 3D geometry it may be necessary to use the results of preliminary simulations to define the feed metal flow path.

The cost function is based on the objective of an optimization problem and is defined as follows:

$$\text{cost} = n \sum_{i=1}^{s_n - 1} p \max[(t_{f_{i+1}} - t_{f_i}), 0] \tag{7}$$

where,

n: Number of user-defined paths to enforce the directional solidification

s_n: Number of points in the n^{th} user-defined path

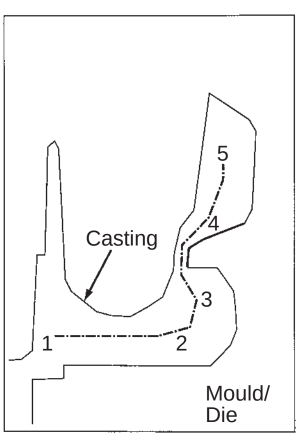
p: Penalty term

t_{f_i}: Freezing time at the i^{th} design point

The freezing time at any point is the time at which the temperature reaches the solidus temperature. The nodal freezing times are computed during the solidification analysis as follows. The freezing times at the design points, then, can be interpolated using the shape functions.

$$t_{f_{i-\text{node}}} = t - \Delta t \left[\frac{T^t_{i-\text{node}} - T_{\text{sol}}}{T^t_{i-\text{node}} - T^{t-\Delta t}_{i-\text{node}}} \right] \tag{8}$$

where



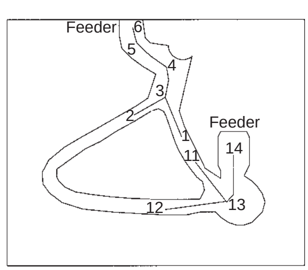


Figure 1. User-defined feed metal flow paths. The figure on the left is a schematic diagram of an axisymmetric casting example and the right is a 2D casting example

$t_{f_{i-\text{node}}}$: Freezing time at the i^{th} node
t: Analysis time (This should not be confused with the computer CPU time.)
Δt: Time step
$T^{t}_{i-\text{node}}$: Nodal temperature at time t
T_{sol}: Solidus temperature.

We have chosen to optimize the die coating thickness. Die coating thickness directly influences the interfacial heat transfer coefficients early in the cooling interval. The correlation proposed by the authors (Lewis and Ransing, 1998) offers a natural choice for the design variables for the optimal die coating thickness problem. The three coefficients viz. a_1, a_2 and a_3 in the correlation (equation 6) are the design variables in the optimization process. Although, the interfacial heat transfer coefficients are a function of temperature, these three coefficients remain constant during the transient analysis. Therefore, they offer the best choice for the design variables.

The final component in optimization is the sensitivity analysis. The finite difference sensitivity approach was adopted due to its simplicity and ease in the implementation.

$$\frac{\partial \text{cost}}{\partial a_i} = \frac{\text{cost}(a_i + \Delta a_i) - \text{cost}(a_i)}{\Delta a_i} \tag{9}$$

As the optimization problem defined is an unconstrained optimization (although, with upper and lower limits on the design variables) the standard quasi-Newton method (BFGS) (NAG, 1988) was employed. It was observed during the analysis that the optimization results were sensitive to the scaling of the design variables. Further research work is necessary to constrain the design variables so that the temporal variation of the heat transfer coefficients will always remain within practically achievable bounds.

4. Example: an axisymmetric aluminium alloy wheel casting

The optimization process has been validated on the gravity die casting example shown in Figure 1. Figure 2 shows the finite element mesh that was used. The cast metal is an aluminium alloy LM25 with 615°C and 550°C liquidus and solidus temperatures respectively. The mould is a steel mould with H13 specification. The initial temperature for the melt and mould were assumed to be 625°C and 150°C. The convection boundary condition of 75 W/m^2°C was applied on the outer surfaces and the ambient temperature was assumed to be 25°C. Constant conductivity values of 186.3 W/m°C and 33.9 W/m°C and density 2790kg/m^3 and 7721kg/m^3 was assumed for the metal and mould respectively. The temperature dependent enthalpy curve used has been tabulated (Table I).

The next step is the division of the metal mould interface to identify the total number of design variables. Along each interface division, the variation of

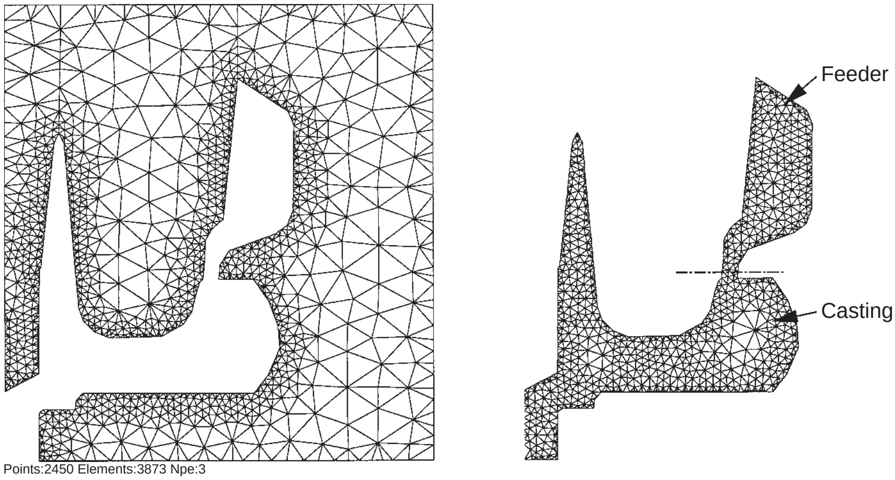


Figure 2. Mould and casting mesh for the axisymmetric wheel casting simulation

Temperature (°C)	Enthalpy (J/Kg)
Metal	
0	0
550	$6.011*10^5$
615	$1.0699*10^6$
800	$1.2721*10^6$
Mould	
0	0
200	$1.424*10^5$
400	$2.488*10^5$
800	$5.696*10^5$

Table I. Enthalpy curve used for the metal and mould

interfacial heat transfer coefficients is same i.e. each division corresponds to three design variables $a1$, $a2$ and $a3$ (Equation 6). The criterion for the division of interface is mostly heuristic and problem specific. It depends on the length of the insulation required, shape of the casting, manufacturing constraints as well as the cost of manufacturing.

Figures 5 and 6 show the division of interface for this problem. The metal-mould interface has been divided into six parts identified as location 1 to 6. The location 1 in Figure 5 corresponds to the shape of the core. The base (location 2) and side (location 3) of the casting are separate subdivisions. It should be noted that with very few number of divisions, the solution may not converge e.g. if there is only one subdivision, the interfacial heat transfer coefficient variation will be the same everywhere and the solution may not converge. On the other hand, too great a number of divisions would generate a large number of design variables which would make the computational cost prohibitive. Also, it may not be practically feasible.

Figures 3 and 4 summarise the optimisation results. Initially, spatially constant, but temporally varying interfacial heat transfer coefficients were applied. Clearly, this represents a traditional model and a hot spot was predicted inside the casting (Figure 3). However, after the optimisation process, the output specification of spatially as well as temporally varying interfacial heat transfer coefficients are shown in Figure 4. It predicts a high initial heat transfer coefficient value ($11kW/m^2C$), almost like a perfect contact, at the base of the casting (location 3, Figure 5) and at the base of the feeder (location 5, Figure 6). Insulation is suggested at location 1, 2 and 6 which corresponds to core, side of the casting and the feeder. The value predicted at the side of the

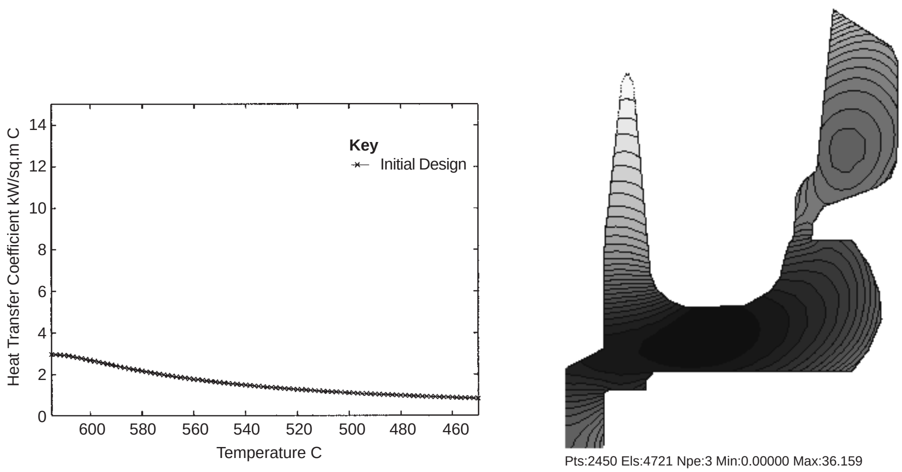


Figure 3.
Initial specification of interfacial heat transfer coefficients at all the segments and the corresponding freezing time contours

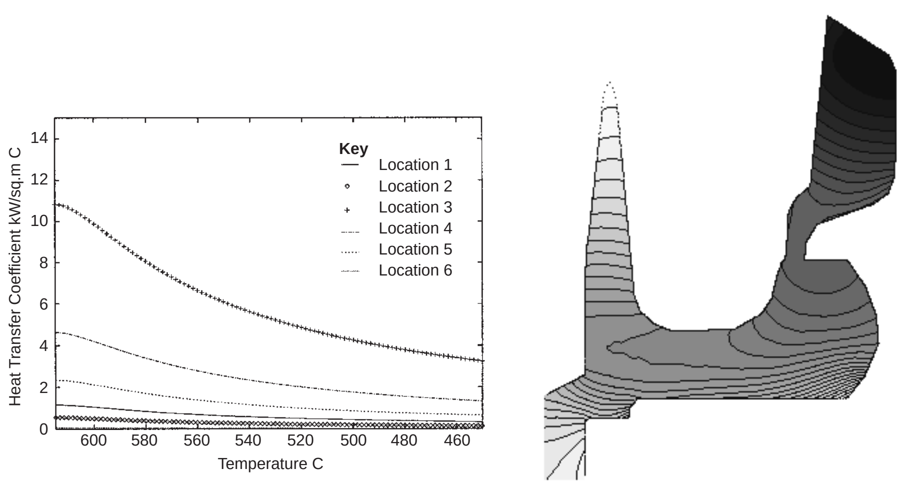


Figure 4.
Final (optimal) specification of interfacial heat transfer coefficients at the respective interface locations and the corresponding freezing time contours. (Refer to Figures 5 and 6 for interface locations)

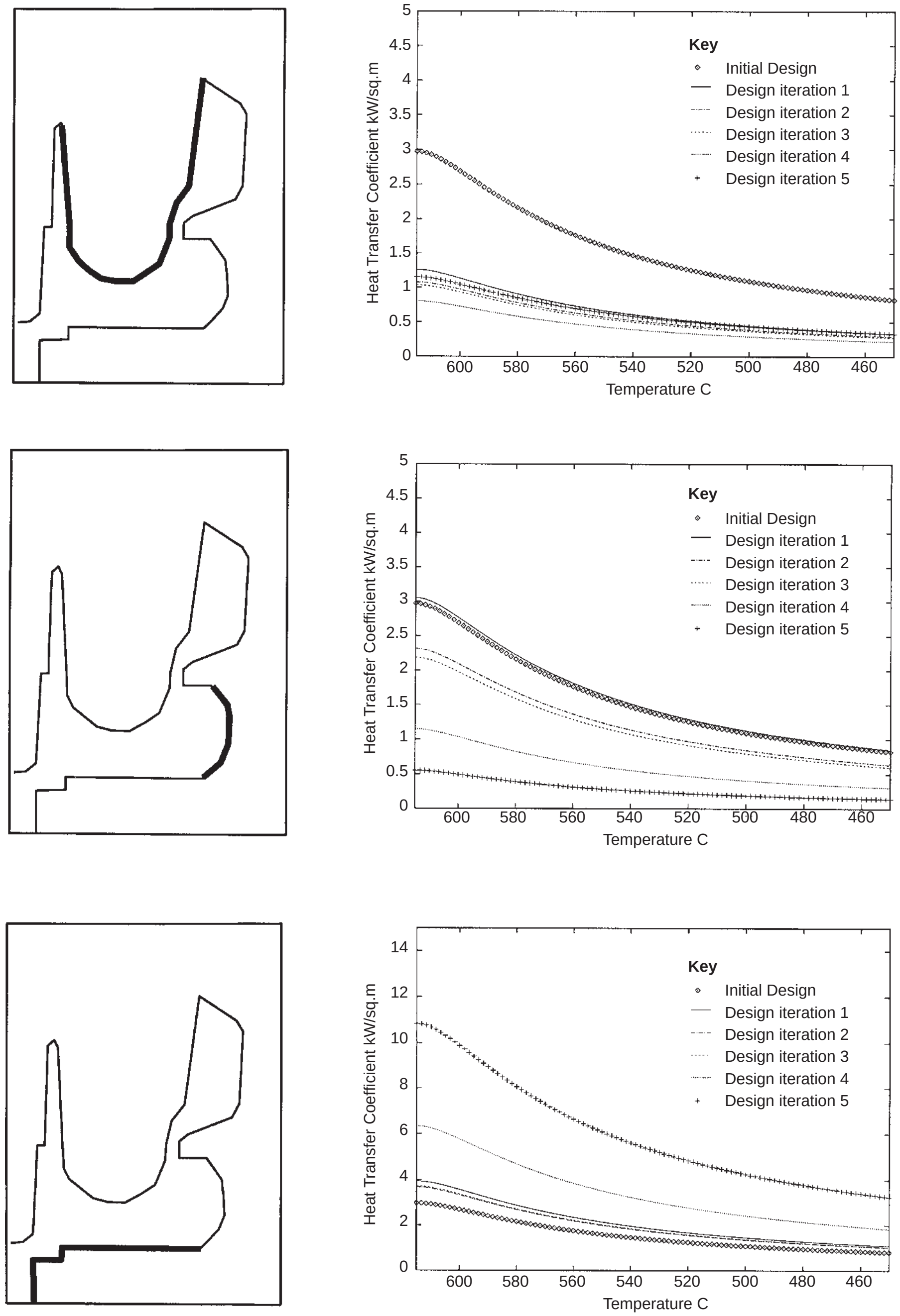


Figure 5.
Location of interface segments referred to as location 1, 2 and 3 from top to bottom and the corresponding variation of interfacial heat transfer coefficients through design iterations

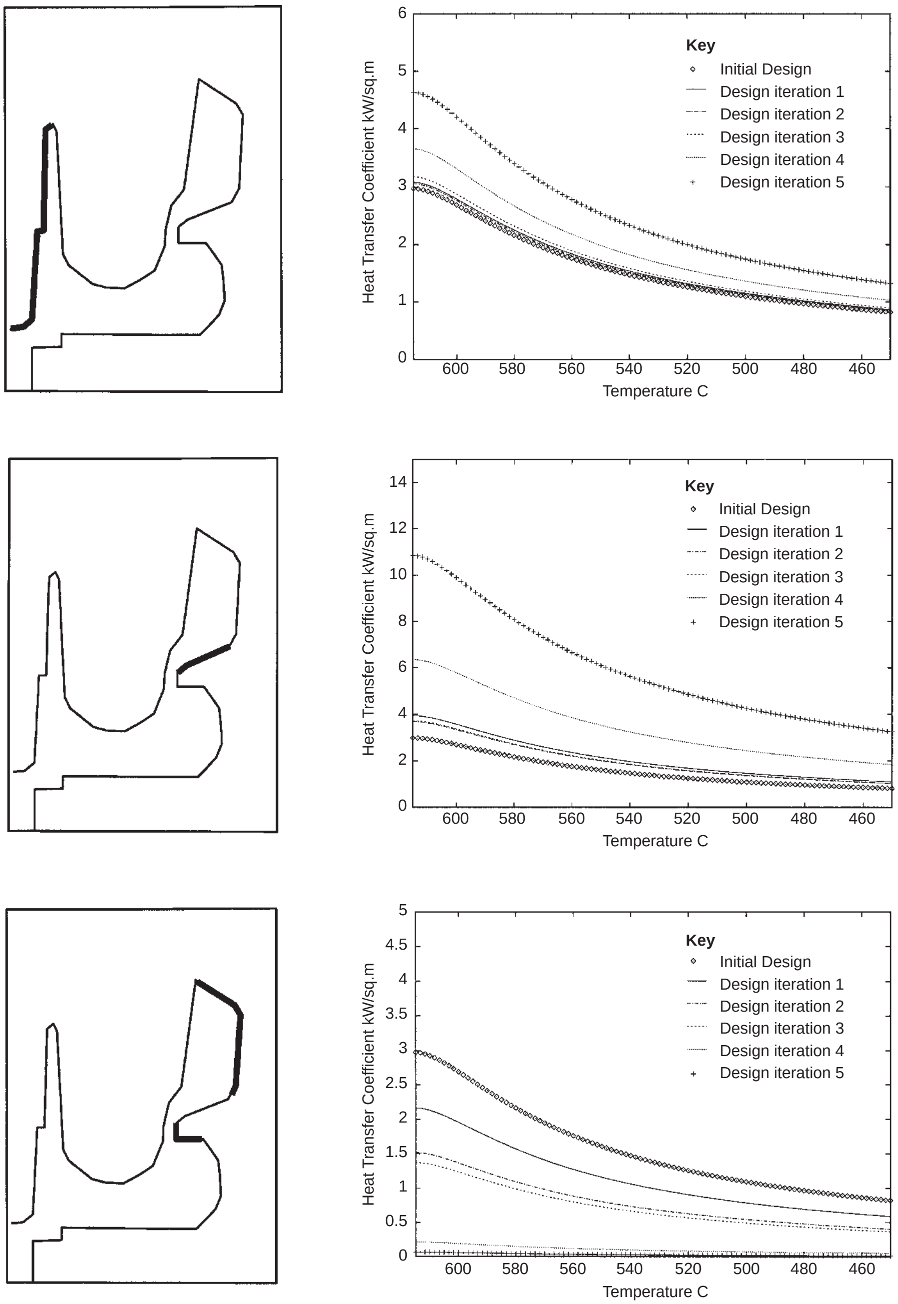


Figure 6.
Location of interface segments referred to as location 4, 5 and 6 from top to bottom and the corresponding variation of interfacial heat transfer coefficients through design iterations

casting (location 2, Figure 5) is around $4.5kW/m^2C$ which is also plausible. At location 4, it suggests an initial value of around $4.5kW/m^2C$. Clearly, by selecting suitable die coating thickness, these values can be realised in practice.

Figures 5 and 6 show the variation in the design variables i.e. the interfacial heat transfer coefficients during each design iteration. The corresponding changes in the solidification pattern from the initial design to the final design are shown in the Figure 7. It can be seen that for the initial design, the cost

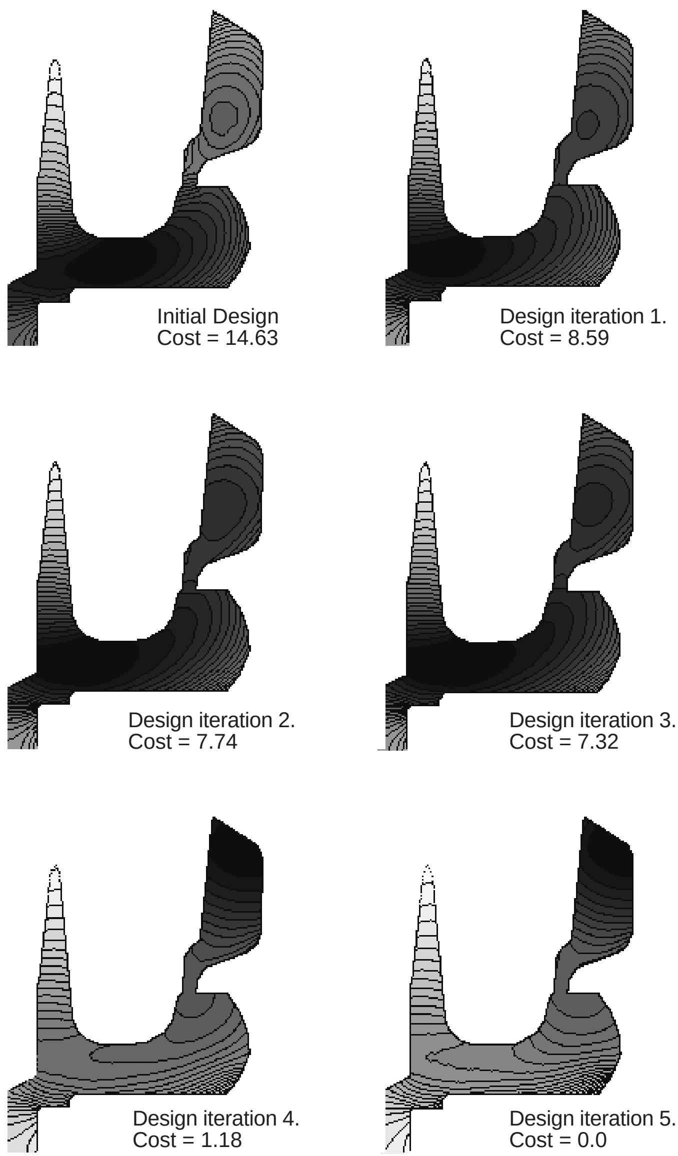


Figure 7.
Changes in the freezing time contours along with cost from the initial design to the optimal design

(equation 7) is 14.63. The cost was minimised in five design iterations. However, it should be noted that the cost may not always become zero. A value close to zero is also acceptable.

5. Example: 2D aluminium alloy casting

Figure 8 shows the finite element mesh that was used for this casting. The cast metal is again the aluminium alloy LM25 and the mould is H13 steel. The

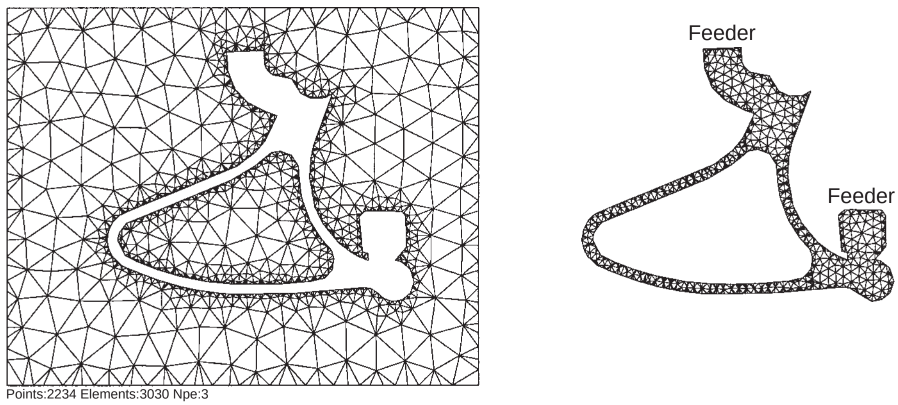


Figure 8.
Mould and casting mesh for the 2D casting simulation

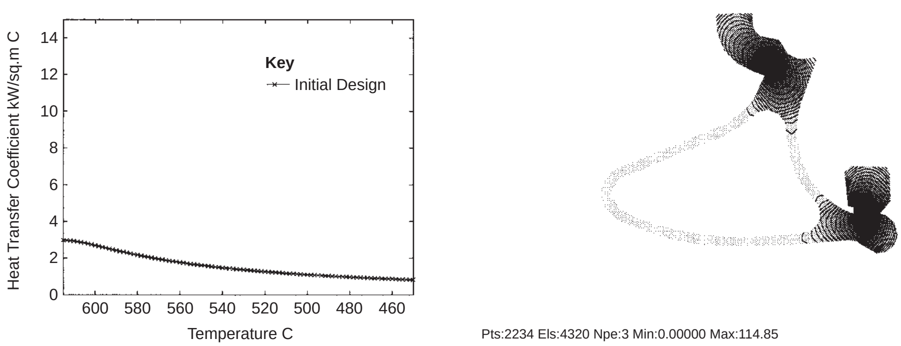


Figure 9.
Initial specification of interfacial heat transfer coefficients at all the segments and the corresponding freezing time contours

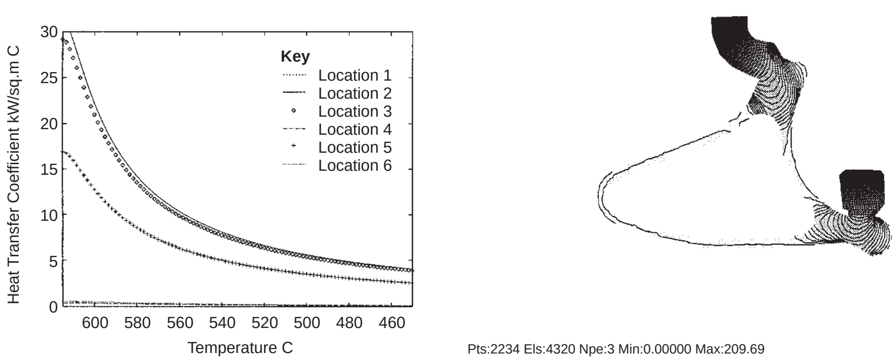


Figure 10.
Final (optimal) specification of interfacial heat transfer coefficients at the respective interface locations and the corresponding freezing time contours. (Refer to Figures 11 and 12 for interface locations)

material properties and boundary conditions used are the same as those in the previous example.

The casting is designed with two feeders. Figure 13 presents the final temperature contours and illustrates that for the initial design with same interfacial heat transfer coefficients along the interface boundary, two potential locations exist for the hot spots. Both are near to the end of the feeders. The objective is to move both the hot spots into the respective feeders and at the same time make sure that the thin section does not freeze off prematurely.

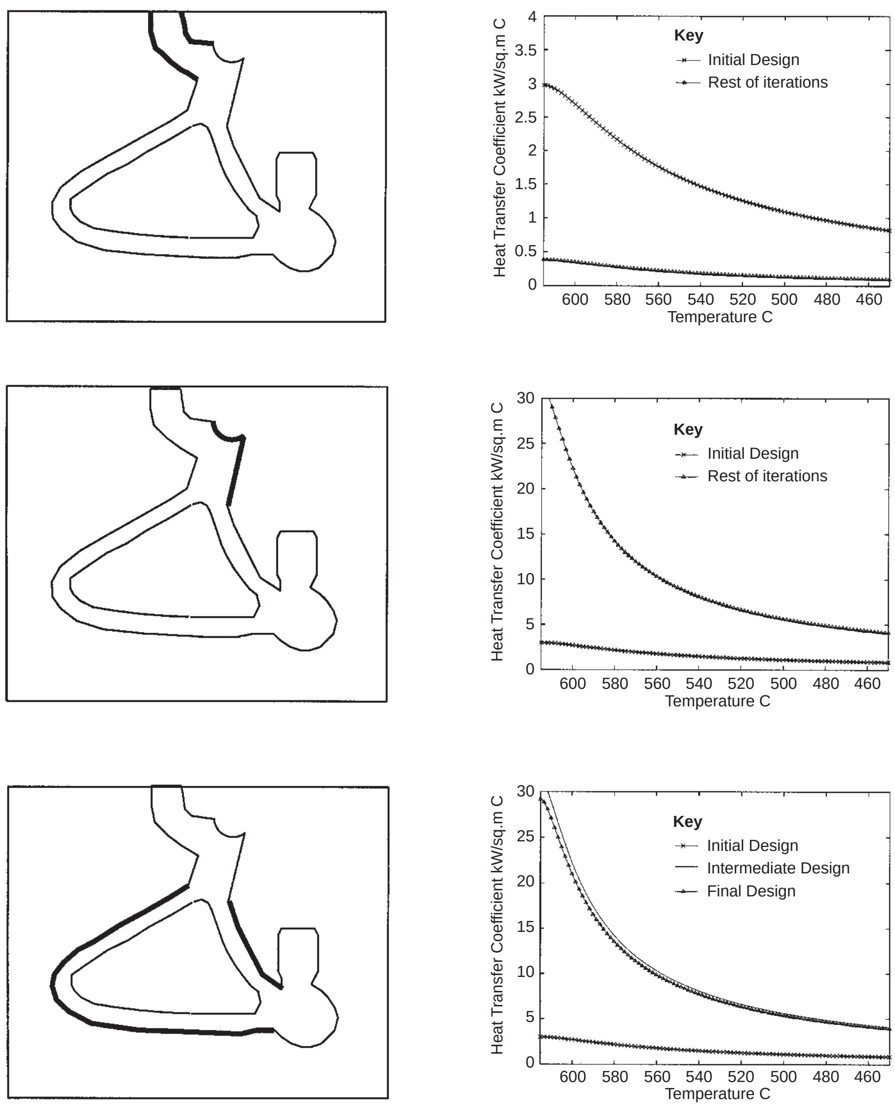


Figure 11. Location of interface segments referred to as location 1, 2 and 3 from top to bottom and the corresponding variation of interfacial heat transfer coefficients through design iterations

For this geometry also, the interface boundary has been subdivided into six divisions (Figures 11 and 12). Two feeders constitute two divisions (locations 1 and 4). The inside (location 6) and outside (location 3) formed two divisions. Remaining parts formed two separate divisions (location 2 and 4). As explained in the previous example, the metal-mould heat transfer at all points in one location (or interface division) is constrained to the same value.

Figures 9 and 10 summarise the optimisation results. Initially, spatially constant but temporally varying interfacial heat transfer coefficients were applied. The optimal specification of spatially as well as temporally varying

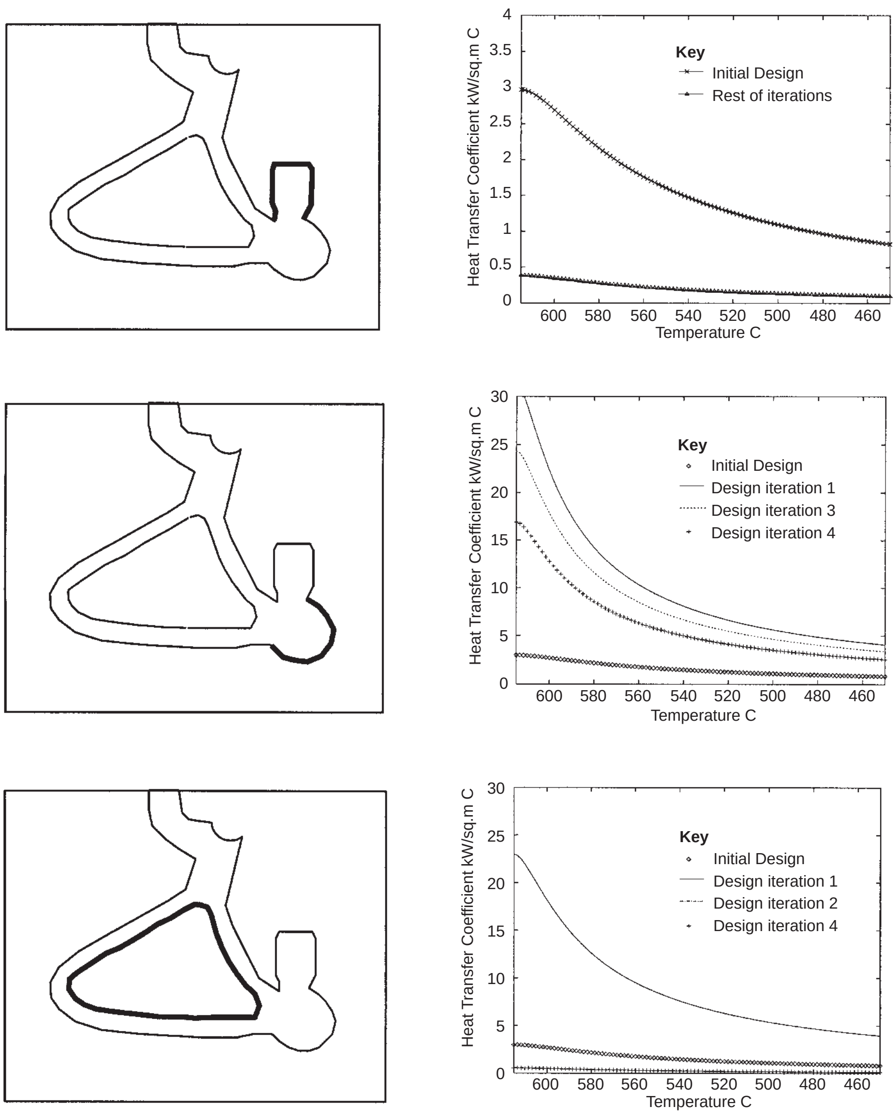


Figure 12.
Location of interface segments referred to as location 4, 5 and 6 from top to bottom and the corresponding variation of interfacial heat transfer coefficients through design iterations

interfacial heat transfer coefficients has been shown in Figure 10. The interfacial heat transfer coefficient variation during design iterations has been shown in Figures 11 and 12. Changes in the freezing time contours from the initial design to the final design have been shown in Figure 13.

This example predicts very high values ($30 kW/m^2$C) at locations 2 and 3. Such high values can not be realised in practice. This may mean that the given feed metal flow path can not be realised by controlling the interface heat transfer coefficients alone. May be the casting would require a greater number

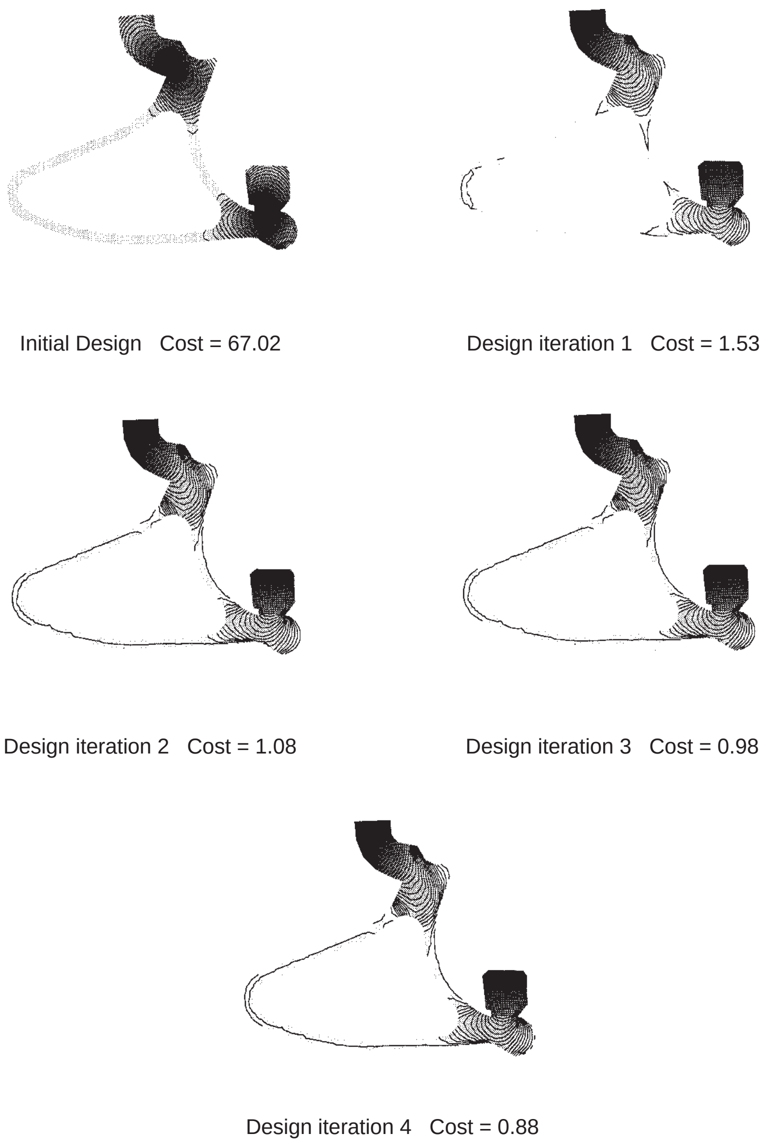


Figure 13.
Changes in the freezing time contours along with cost from the inital design to the optimal design

of feeders. It is also possible that the metal-mould interface needs to be further sub-divided to generate a greater number of design possibilities. Certainly, more work needs to be done in constraining the design variables, so that the optimisation program would always predict plausible results.

6. Conclusion

The use of optimisation techniques for the optimal design of feeder size and shape has already been demonstrated (Tortorelli *et al.*, 1994 and Morthland *et al.*, 1995). However many feeding design decisions such as placement of chills, insulation, exothermic/endothermic pads or die coating thickness etc. are taken with the aid of numerical simulation of the solidification process. Mostly, such decisions are taken based on a trial and error method.

Generally, the influence of chills, insulation etc. in the numerical simulation is considered in conjunction with appropriate values for the interfacial heat transfer coefficients. It is known that these values vary temporally as well as spatially and hence, it was difficult to find optimal values with optimisation techniques. The Lewis-Ransing correlation has offered a natural choice for the selection of design variables during an optimal feeding design process. The constants $a1$, $a2$ and $a3$ in the correlation have been used as design variables to optimise the freezing time contours. The results have been demonstrated on two representative examples.

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